

Class: XII

Monthly Exam (Sep-2019)

Marks: 50

Time: 1 hrs 30 minutes

Mathematics(041)

Date: 23\09\2019

SECTION-A

Question numbers 1 to 10 carry 1 mark each.

- For what value of 'x', the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?
- The function $f(x) = \frac{4-x^2}{4x-x^3}$
 - discontinuous at only one point
 - discontinuous exactly at two points
 - discontinuous exactly at 3 points
 - none
- Given $f(x) = 4x^8$, then
 - $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$
 - $f\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$
 - $f\left(-\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$
 - $f\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$
- If a cone of maximum volume is inscribed in a given sphere, then the ratio of the height of the cone to the diameter of the sphere is ---
 - 3/4
 - 1/3
 - 1/4
 - 2/3
- The function $f(x) = \frac{x}{1+|x|}$ is
 - strictly increasing
 - constant function
 - identity function
 - none
- Find the slope of the normal to the curve $y = x + \sin x$, at $x = \frac{\pi}{2}$.

7. Evaluate: $\int \frac{\sin x}{\cos^2 x} dx$
8. Evaluate: $\int \frac{1}{4+9x^2} dx$
9. Evaluate: $\int \frac{\log x}{x} dx$
10. Find the anti-derivative of $\cos 2x$.

SECTION-B

Question numbers 11 to 15 carry 2 marks each.

11. Find the product : $\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{pmatrix}$.
12. Find the rate of change of volume of a sphere with respect to its surface area when the radius is 2 cm.
13. If the tangent to the curve $y = x^3 + ax + b$ at (1, -6) is parallel to the line $x - y + 5 = 0$, find the values of 'a' and 'b'.
14. Find the maximum and minimum values of the function $9x^2 + 12x + 2$.
15. If $y = A \sin x + B \cos x$, then prove that $\frac{d^2 y}{dx^2} + y = 0$

SECTION-C

Question numbers 16 to 18 carry 4 marks each.

16. If $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$, find A^{-1} .
17. If $f(x) = \begin{cases} x^2 + 3x + a, & \text{for } x \leq 1 \\ bx + 2, & \text{for } x > 1 \end{cases}$ is everywhere differentiable, find the values of a and b . **(OR)**

Verify Rolle's theorem for the function $f(x) = e^x(\sin x - \cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

18. Integrate: $\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$ (OR)

Integrate: $\int \frac{1}{\cos(x+a)\cos(x+b)} dx$

SECTION-D

Questions 19 to 21 carry 6 marks each.

19. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ (Volume of the sphere). (OR)

Find the local maximum and local minimum, if any, of the function

$$f(x) = \sin^4 x + \cos^4 x, \quad 0 < x < \frac{\pi}{2}.$$

20. Find: $\int \sin^4 x dx$ (OR) $\int \frac{x^3}{x+2} dx$

21. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1} and hence solve the system of linear

equations $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$.